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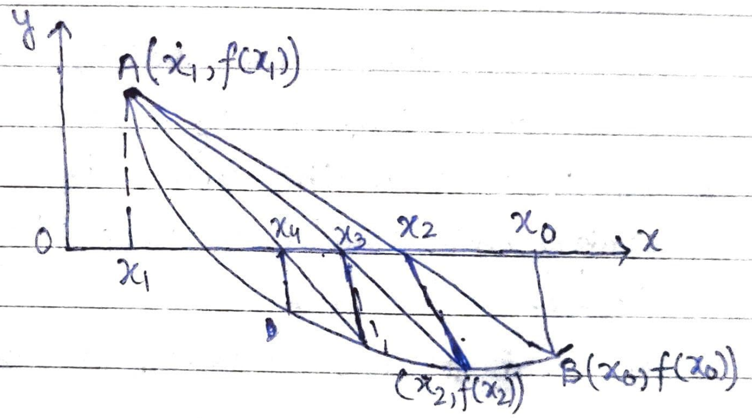
Regula-Falsi Method / Method of False Proportion

In this method we take two points x_0 and x_1 such that $f(x_0)$ and $f(x_1)$ are opposite signs, i.e., $f(x_1)f(x_0) < 0$. This implies that the graph of $y = f(x)$ will surely cut the axis of 'x' b/w $x = x_0$ and $x = x_1$. That is a root will lie b/w x_0 and x_1. In this method, the part of the curve $y = f(x)$ b/w the points $[x_0, f(x_0)]$ & $[x_1, f(x_1)]$ is replaced by the chord joining these points. So the equation of this chord is given by

$$\frac{y - f(x_0)}{x - x_0} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$



The chord intersects the x-axis b/w the points where $y = 0$. Thus we get $x = x_0 - \frac{f(x_0)(x_1 - x_0)}{f(x_1) - f(x_0)}$

The intersection of the chord with the x-axis gives an approximation to the root. Hence, the second approximation to the root is given by

$$x_2 = x_0 - \frac{f(x_0)(x_1 - x_0)}{f(x_1) - f(x_0)}$$

If $f(x_0)$ and $f(x_2)$ are of opposite signs, then root will lie b/w 'x_0' and 'x_2'. Now replacing the part of curve b/w the points $(x_0, f(x_0))$ & $(x_2, f(x_2))$ by the chord joining these points & this chord intersects the x-axis, where we get next approximation to the root given by

$$x_3 = x_0 - \frac{f(x_0)(x_2 - x_0)}{f(x_2) - f(x_0)}$$

Continuing this process until we get a root of desired accuracy i.e., $x_{n+1} = x_{n-1} - \frac{f(x_{n-1})(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}$

Note :-

1) We choose such points x_0 & x_1 for which $f(x_0) f(x_1) < 0$

2) We choose the points x_0 & x_1 such that they form sufficiently small interval, In this interval the curve is considered straight line, i.e., it is called Regula-Falsi Method.

3) The straight line is called a variable chord.

Secant Method

It is an improvement on Regula-Falsi Method as it does not require the condition that $f(x_0) . f(x_1) < 0$. In this method the graph of $y = f(x)$ is approximated by a secant line at each iteration. Secant line is nothing but the chord joining the initial limits of the interval. Taking x_0, x_1 as the initial limits of the interval, the equation of the chord joining the points $(x_0, f(x_0))$ and $(x_1, f(x_1))$ as

$$y - f(x_1) = \frac{f(x_1) - f(x_0)}{(x_1 - x_0)}(x - x_1)$$

This secant line cuts the x-axis i.e., $y=0$, then the abscissa of the point is given by

$$x_2 = x_1 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_1)$$

which is an approximation to the root. The successive approximation is given by

$$x_{n+1} = x_n - \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} f(x_n), \quad n \geq 1$$

Note :

1) In this method, $f(x_0)$ & $f(x_1)$ are not necessarily of opposite signs.
2) It is not necessary that the interval (x_0, x_1) must contain the root.
3) This method fails if any $f(x_n) = f(x_{n-1})$
4) The rate of convergence is faster than that of Regula - Falsi Method.
5) The order of convergence of secant method is 1.62.

Ques) Find a real root of the equation $f(x) = x^3 - 2x - 5 = 0$ by Regula-Falsi Method upto three decimal places.

Soln) $\because f(x) = x^3 - 2x - 5 = 0$ ——— (1)

$f(0) = -5 < 0$, $f(1) = -6 < 0$

$f(2) = 8 - 4 - 5 = -1 < 0$

$f(3) = 27 - 6 - 5 = 16 > 0$

$\therefore$ The root lies between '2' and '3'.

$\therefore$ Taking $x_0 = 2$, $x_1 = 3$, $f(x_0) = -1$, $f(x_1) = 16$ then by Regula-falsi Method we get

$$x_2 = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} \cdot f(x_0)$$

$$\Rightarrow x_2 = 2 - \frac{(3-2)\cdot(-1)}{16+1}$$

$$= 2 + \frac{1}{17} = 2.0588$$

Now,

$$f(x_2) = f(2.0588)$$

$$= (2.0588)^3 - 2(2.0588) - 5$$

$$= -0.3911 < 0$$

Now the root lies b/w 2.0588 & 3

taking $x_0 = 2.0588, x_1 = 3, f(x_0) = -0.3911$

$f(x_1) = 16$

$$\Rightarrow x_3 = 2.058 - \frac{(3-2.0588)}{16+0.3911} \times (-0.3911)$$

$$= 2.0813$$

$$\Rightarrow f(x_3) = (2.0813)^3 - 2(2.0813) - 5$$

$$= -0.1468$$

$\therefore$ The root lies b/w 2.0813 & 3

taking $x_0 = 2.0813, x_1 = 3, f(x_0) = -0.1468, f(x_1) = 16$

$$\Rightarrow x_4 = 2.0813 - \frac{(3-2.0813)}{(16+0.1468)} \times (-0.1468)$$

$$= 2.0897$$

$$\Rightarrow f(x_4) = (2.0897)^3 - 2(2.0897) - 5$$

$$= -0.054$$

$\therefore$ the root lies b/w 2.0897 & 3

taking $x_0 = 2.0897, x_1 = 3, f(x_0) = -0.054, f(x_1) = 16$

$$\Rightarrow x_5 = 2.0897 - \frac{(3-2.0897)}{16+0.054} \times (-0.054)$$

$$= 2.0928$$

$$\Rightarrow f(x_5) = (2.0928)^3 - 2(2.0928) - 5 = -0.0195$$

$\therefore$ The root lies b/w 2.0928 & 3

taking $x_0 = 2.0928$, $x_1 = 3$, $f(x_0) = -0.0195$, $f(x_1) = 16$

$$\Rightarrow x_6 = 2.0928 - \frac{(3-2.0928)}{16+0.0195} \times (-0.0195)$$

$$= 2.0939$$

$$\Rightarrow f(x_6) = (2.0939)^3 - 2(2.0939) - 5$$

$$= -0.0074$$

Thus the root lies b/w 2.0939 & 3

Taking $x_0 = 2.0939$, $x_1 = 3$, $f(x_0) = -0.0074$, $f(x_1) = 16$

$$\Rightarrow x_7 = 2.0939 - \frac{(3-2.0939)}{(16+0.0074)} (-0.0074)$$

$$= 2.0943$$

$$\Rightarrow f(x_7) = (2.0943)^3 - 2(2.0943) - 5$$

$$= -0.0028$$

$\therefore$ the root lies b/w 2.0943 & 3

Taking $x_0 = 2.0943$, $x_1 = 3$, $f(x_0) = -0.0028$, $f(x_1) = 16$

$$x_8 = (2.0943) - \frac{(3-2.0943)}{16+0.0028} (-0.0028)$$

$$= 2.0945$$

$\therefore$ the root of given equation upto three decimal places is 2.094.

Ans